Chapter 5
Principles of Force Convection

5.1 Introduction

The preceding chapters have considered the mechanism and calculation of conduction heat transfer. Convection was considered only insofar as it related to the boundary conditions imposed on a conduction problem. We now wish to examine the methods of calculating convection heat transfer and, in particular, the ways of predicting the value of the convection heat-transfer coefficient *h*.

Our development in this chapter is primarily analytical in character and is concerned only with forced-convection flow systems. Subsequent chapters will present empirical relations for calculating forced-convection heat transfer and will also treat the subjects of natural convection.

5.2 Viscous Flow
5.2.1 Flow on a flat plate (external flow)

Consider the flow over a flat plate as shown in Figures 5.1 and 5.2. Beginning at the leading edge of the plate, a region develops where the influence of viscous forces is felt. These viscous forces are described in terms of a shear stress τ between the fluid layers. If this stress is assumed to be proportional to the normal velocity gradient, we have the defining equation for the viscosity,

$$\tau = \mu \frac{du}{dy} \qquad 5.1$$

The region of flow that develops from the leading edge of the plate in which the effects of viscosity are observed is called the *boundary layer*. Some arbitrary point is used to designate the *y* position where the boundary layer ends; this point is usually chosen as the *y* coordinate where the velocity becomes 99 percent of the free-stream value.

Initially, the boundary-layer development is laminar, but at some critical distance from the leading edge, depending on the flow field and fluid properties, small disturbances in the flow begin to become amplified, and a transition process takes place until the flow becomes turbulent. The turbulent-flow region may be pictured as a random churning action with chunks of fluid moving to and fro in all directions.
The transition from laminar to turbulent flow occurs when

$$\frac{u_\infty x}{\upsilon} = \frac{\rho u_\infty x}{\mu} = Re > 5 * 10^5 \quad \text{(transiant)}$$

$$Re \le 5 * 10^5 \quad \text{(laminar)}$$

Where

u_∞= free stream velocity, m/s.

x = distance from leading edge, m.

$\upsilon = \frac{\mu}{\rho}$ =kinematic viscosity, m^2/s.

$$Re_x = \frac{u_\infty x}{\upsilon} \qquad 5.2$$

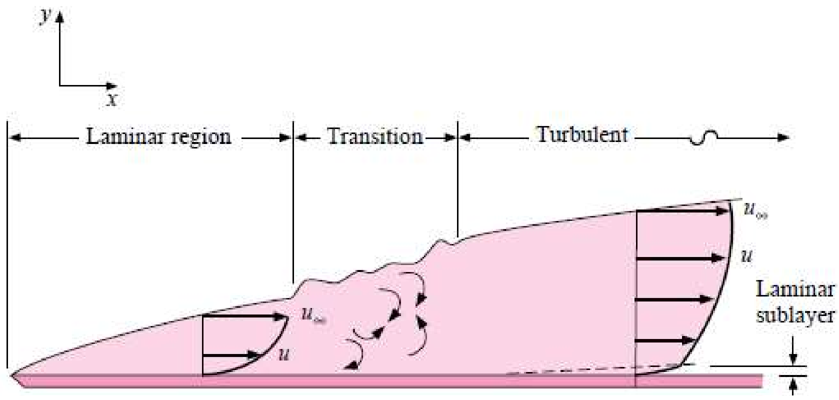


Figure 5.1: Sketch showing different boundary-layer flow regimes on a flat plate.

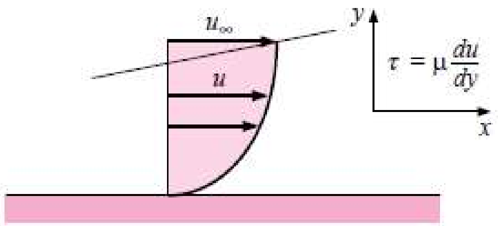


Figure 5.2: Laminar velocity profile on a flat plate.

5.3 Laminar Boundary Layer on a Flat Plate

Consider the elemental control volume shown in Figure 5.3. We derive the equation of motion for the boundary layer by making a force-and-momentum balance on this element.

To simplify the analysis we assume:

1. The fluid is incompressible and the flow is steady.

2. There are no pressure variations in the direction perpendicular to the plate.

3. The viscosity is constant.

4. Viscous-shear forces in the *y* direction are negligible.

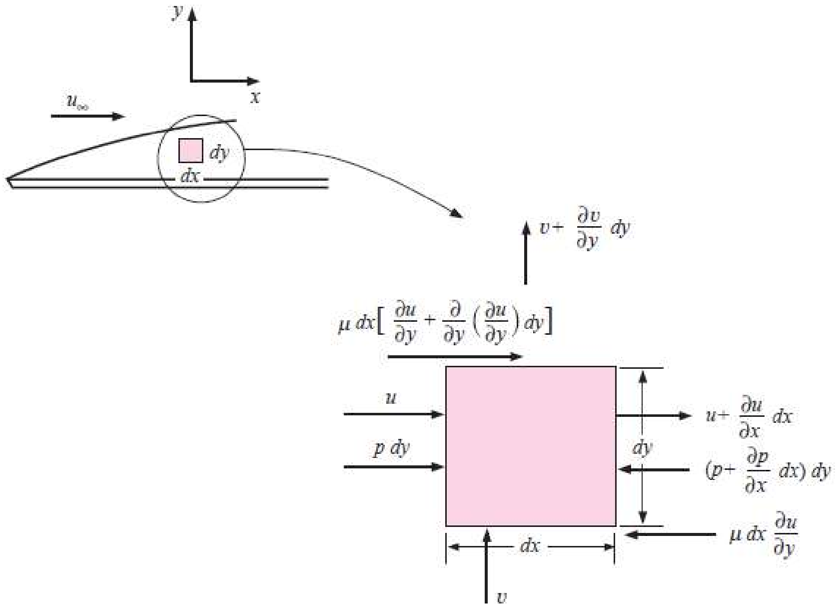


Figure 5.4 Elemental control volume for force balance on laminar boundary layer.

The mass continuity equation for the boundary layer.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad 5.3$$

The momentum equation of the laminar boundary layer with constant properties.

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} \qquad 5.4$$

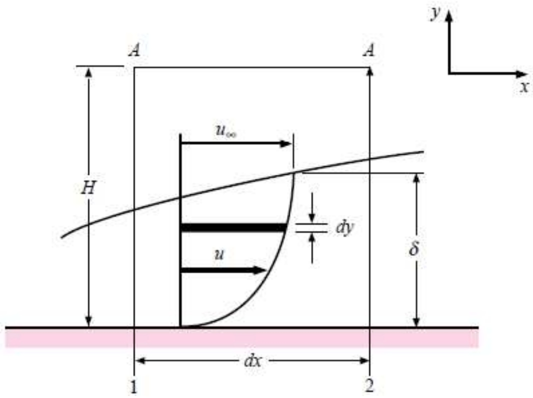


Figure 5.5: Elemental control volume for integral momentum analysis of laminar boundary layer.

The mass flow through plane 1 in Figure 5.5 is

$$\dot{m} = \int_0^H \rho u \, dy$$

Von Kármán approximate solution of equations 5.3 and 5.4 gives

$\delta = \frac{4.64x}{Re_x^{1/2}}$

The exact solution of the boundary-layer equations

$$\delta = \frac{5x}{Re_x^{1/2}} \qquad 5.5$$

The velosity profile of the stream in x-direction within the baundary layer is given by:

$$\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad 5.6$$

And the energy equation of the laminar boundary layer is:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad 5.7$$

For low velocity incompressible flow, we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad 5.8$$

There is a striking similarity between equation 5.8 and the momentum equation for constant pressure,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \alpha\frac{\partial^2 u}{\partial y^2} \qquad 5.9$$

Example 5.1: Mass Flow and Boundary-Layer Thickness

Air at 27◦C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the boundary-layer thickness at distances of 20 cm and 40 cm from the leading edge of the plate. Calculate the mass flow that enters the boundary layer between $x = 20$ cm and $x = 40$ cm. The viscosity of air at 27◦C is 1.85×10^{-5} kg/m· s. Assume unit depth in the *z* direction.

Solution

The density of air is calculated from

$\rho = \frac{p}{RT} = \frac{1.0132*10^5}{(287)(27+273)} = 1.177$ kg/m^3

$Re_x = \frac{u_\infty x}{v}$

At $x = 20\ cm$: $Re_x = \frac{u_\infty x}{v} = \frac{\rho u_\infty x}{\mu} = \frac{1.177*2*0.2}{1.85\times10-5} = 25{,}448$

At $x = 40\ cm$: $Re_x = \frac{\rho u_\infty x}{\mu} = \frac{1.177*2*0.4}{1.85\times10-5} = 50{,}896$

The boundary layer thickness is calculated from:

$$\delta = \frac{4.64x}{Re_x^{1/2}}$$

At $x = 20\ cm$, $\delta = \frac{4.64*0.2}{(25488)^{1/2}} = 0.00582$ m.

At $x = 40\ cm$, $\delta = \frac{4.64*0.4}{(50896)^{1/2}} = 0.00823$ m.

To calculate the mass flow that enters the boundary layer from the free stream between $x = 20$ cm and $x = 40$ cm, we simply take the difference between the mass flow in the boundary layer at these two *x* positions. At any *x* position the mass flow in the boundary layer is given by the integral

$$\dot{m} = \int_0^{\delta} \rho u\, dy$$

$$u = u_\infty \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

$$\dot{m} = \int_0^{\delta} \rho u_\infty \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\ dy = \frac{5}{8}\rho u_\infty \delta$$

$$\Delta\dot{m} = \frac{5}{8}\rho u_\infty(\delta_{40} - \delta_{20}) = \frac{5}{8}(1.177 * 2(0.00823 - 0.00582) = 0.00354\ \text{ kg/s}$$

5.4 Thermal boundary layer

Just as the hydrodynamic boundary layer was defined as that region of the flow where viscous forces are felt, a thermal boundary layer may be defined as that region where temperature gradients are present in the flow. These temperature gradients would result from a heat-exchange process between the fluid and the wall.

Consider the system shown in Figure 5.6. The temperature of the wall is T_w, the temperature of the fluid outside the thermal boundary layer is T_∞, and the thickness of the thermal boundary layer is designated as δ_t . At the wall, the velocity is zero, and the heat transfer into the fluid takes place by conduction. Thus the local heat flux per unit area is:

$$\frac{q}{A} = -k\frac{\partial T}{\partial y}\bigg]_{wall} \qquad 5.10$$

From Newton's law of cooling

$$\frac{q}{A} = h(T_w - T_\infty) \qquad 5.11$$

where h is the convection heat-transfer coefficient. Combining these equations, we have

$$h = \frac{-k\frac{\partial T}{\partial y}\Big]_{wall}}{(T_w - T_\infty)} \qquad 5.12$$

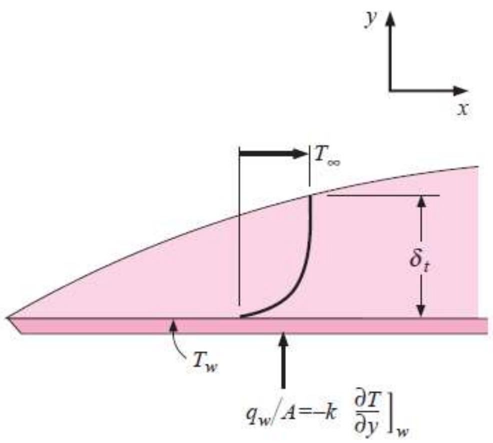
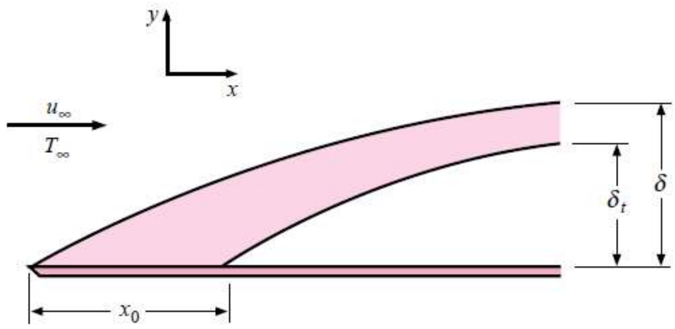


Figure 5.6: Temperature profile in the thermal boundary layer.

Then we need only find the temperature gradiant at the wall to evaluate h. Therefore, the temperature distribution is:

$$\frac{\theta}{\theta_\infty} = \frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 \qquad 5.13$$



The thermal boundary layer can calculated from the equation below:

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}\left[1 - \left(\frac{x_o}{x}\right)^{3/4}\right]^{1/3} \qquad 5.14$$

When the plate is heated over the entire length, $x_o = 0$, and

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3} \qquad 5.15$$

Where the Prandtl number is dimensionless when a consistent set of units is used:

$$Pr = \frac{v}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{c_p \mu}{k} \qquad 5.16$$

The local convective heat transfer coefficient is calculated from the equation as below:

$$h_x = 0.332k\ Pr^{1/3} \left(\frac{u_\infty}{vx}\right)^{1/2} \left[1 - \left(\frac{x_o}{x}\right)^{3/4}\right]^{-1/3} \qquad 5.17$$

The equation may be non-dimensionalized by multiplying both sides by $\frac{x}{k}$, producing the dimensionless group on the left side,

$$Nu_x = \frac{h_x x}{k} \qquad 5.18$$

called the Nusselt number after Wilhelm Nusselt, who made significant contributions to the theory of convection heat transfer. Finally,

$$Nu_x = 0.332\ Pr^{1/3} Re_x^{1/2} \left[1 - \left(\frac{x_o}{x}\right)^{3/4}\right]^{-1/3} \qquad 5.19$$

or, for the plate heated over its entire length, $x_o = 0$ and

$$Nu_x = 0.332\ Pr^{1/3} Re_x^{1/2} \qquad 0.6 < Pr > 50 \qquad 5.20\ a$$

Equation (5.20 a) is applicable to fluids having Prandtl numbers between about 0.6 and 50. It would not apply to fluids with very low Prandtl numbers like liquid metals or to high- Prandtl-number fluids like heavy oils or silicones. For a very wide range of Prandtl numbers, Churchill and Ozoe have correlated a large amount of data to give the following relation for laminar flow on an isothermal flat plate:

$$Nu_x = \frac{0.3387 Re_x^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}} \qquad Re_x Pr > 100 \qquad 5.20\ b$$

Equations (5.17), (5.19), and (5.20 a) express the local values of the heat-transfer coefficient in terms of the distance from the leading edge of the plate and the fluid properties. For the case where $x_o = 0$ the average heat-transfer coefficient and Nusselt number may be obtained by integrating over the length of the plate:

$$\bar{h} = \frac{\int_0^L h_x dx}{\int_0^L dx} = 2h_{x=L} \qquad 5.21$$

assuming the heated section is at the constant temperature T_w. For the plate heated over the entire length,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 2Nu_{x=L} \qquad 5.22$$

Or

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.664 Re_L^{1/2} Pr^{1/3} \qquad 5.23$$

Where:

$$Re_L = \frac{\rho u_\infty L}{\mu} \qquad 5.24$$

For a plate where heating starts at $x = x_o$, it can be shown that the average heat transfer coefficient can be expressed as

$$\bar{h}_{x_o-L} = h_{x=L}\left(2L\,\frac{1-\left(x_o/L\right)^{3/4}}{L-x_o}\right)$$

In this case, the total heat transfer for the plate would be

$$q_{total} = \bar{h}_{x_o-L}(L - x_o)(T_w - T_\infty)$$

The foregoing analysis was based on the assumption that the fluid properties were

constant throughout the flow. When there is an appreciable variation between wall and free-stream conditions, it is recommended that the properties be evaluated at the so-called *film temperature* T_f , defined as the arithmetic mean between the wall and free-stream temperature,

$$T_f = \frac{T_w + T_\infty}{2} \qquad 5.25$$

Constant Heat Flux

The above analysis has considered the laminar heat transfer from an isothermal surface. In many practical problems the surface *heat flux* is essentially constant, and the objective is to find the distribution of the plate-surface temperature for given fluid-flow conditions. For the constant-heat-flux case it can be shown that the local Nusselt number is given by

$$Nu_x = 0.453\, Re_x^{1/2} Pr^{1/3} \qquad 5.26$$

which may be expressed in terms of the wall heat flux and temperature difference as

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} \quad 5.27$$

Where:

q_w : heat flux, W/m^2

Note that the heat flux $q_w = \frac{q}{A}$ is assumed constant over the entire plate surface.

$$\overline{T_w - T_\infty} = \frac{1}{L}\int_0^L (T_w - T_\infty)dx = \frac{1}{L}\int_0^L \frac{q_w x}{kNu_x}dx$$

$$\overline{T_w - T_\infty} = \frac{q_w L/k}{0.6795 Re_L^{1/2} Pr^{1/3}} \quad 5.28$$

$$q_w = \frac{3}{2}h_{x=L}(T_w - T_\infty)$$

For constant heat flux case and the properties evaluated at the film temperature:

$$Nu_x = \frac{0.4637 Re_x^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.0207}{Pr}\right)^{2/3}\right]^{1/4}} \qquad Re_x Pr > 100 \quad 5.29$$

Example 5.1:

Air at 27◦C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the boundary-layer thickness at distances of 20 cm and 40 cm from the leading edge of the plate. Calculate the mass flow that enters the boundary layer between $x = 20$ cm and $x = 40$ cm. The viscosity of air at 27◦C is 1.85×10^{-5} kg/m· s. Assume unit depth in the *z* direction.

Example 5.2:Isothermal Flat Plate Heated Over Entire Length

For the flow system in Example 5.1 assume that the plate is heated over its entire length to a temperature of 60◦C. Calculate the heat transferred in (*a*) the first 20 cm of the plate and (*b*) the first 40 cm of the plate.

Solution

$T_f = \frac{T_w + T_\infty}{2}$, $T_\infty = 27^\circ C$, $u_\infty = 2$ m/s, T_w=60˚C

$T_f = \frac{60+27}{2} = 43.5 + 273 = 316.5$ K

We find the properties of air at film temperature.

$\upsilon = 17.36 * 10^{-6}$ m^2/s, $Pr = 0.7$, $k = 0.02749$ W/m.˚C.

$c_p = 1.006$ Kj/kg. ˚C.

At $x = 20\ cm$

$Re_x = \frac{u_\infty x}{v} = \frac{2*0.2}{17.36*10^{-6}} = 23041$

$Nu_x = \frac{h_x x}{k}$

$Nu_x = 0.332\ Pr^{1/3} Re_x^{1/2} = 0.332\ (0.7)^{1/3} * (23041)^{1/2} = 44.74$

$h_x = Nu_x \frac{k}{x} = 44.74 * \frac{0.02749}{0.2} = 6.15$ W/m^2. ˚C

The average value of the heat-transfer coefficient is twice this value, or

$\bar{h} = 2h_x = 2 * 6.15 = 12.30$

$q = \bar{h}A(T_w - T_\infty) = 12.3 * (0.2)(60 - 27) = 81.18$ W

At $x = 40\ cm$

$Re_x = \frac{u_\infty x}{v} = \frac{2*0.4}{17.36*10^{-6}} = 46082$

$Nu_x = \frac{h_x x}{k}$

$Nu_x = 0.332\ Pr^{1/3} Re_x^{1/2} = 0.332\ (0.7)^{1/3} * (46082)^{1/2} = 63.28$

$h_x = Nu_x \frac{k}{x} = 63.28 * \frac{0.02749}{0.4} = 4.349$ W/m^2. ˚C

The average value of the heat-transfer coefficient is twice this value, or

$\bar{h} = 2h_x = 2 * 4.349 = 8.698$

$q = \bar{h}A(T_w - T_\infty) = 8.698\ * (0.4)(60 - 27) = 114.8$ W

EXAMPLE 5.3: Flat Plate with Constant Heat Flux

A 1.0-kW heater is constructed of a glass plate with an electrically conducting film that produces a constant heat flux. The plate is 60 cm by 60 cm and placed in an airstream at 27◦C, 1 atm with u_∞ =5 m/s. Calculate the average temperature difference along the plate.

Solution

Properties should be evaluated at the film temperature, but we do not know the plate temperature. for an initial calculation, we take the properties at the free-stream conditions of

At $T_\infty = 27$ ˚C we find the properties of the fluid

$v = 15.96 * 10^{-6}$ m^2/s, $Pr = 0.708$, $k = 0.02624$ W/m.˚C.